Effective Theory of Wilson Lines and Deconfinement

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To study the deconfining phase transition at nonzero temperature, I suggest constructing an effective theory for straight, thermal Wilson lines in three dimensions. This is a gauged, nonlinear sigma model for adjoint matrices, where the temperature naturally provides an ultraviolet cutoff. Especially near the transition, the Wilson line may undergo a Higgs effect in the deconfined phase: as an adjoint field, this can generate eigenvalue repulsion in the effective theory.

Recent results at the Relativistic Heavy Ion Collider (RHIC) demonstrate qualitatively new behavior for the collisions of heavy ions at high energies [1]. RHIC appears to have entered a region above T_c, the temperature for deconfinement, reaching up to temperatures a few times T_c. The experimental results cannot be explained if the transition is directly from a confined phase to a perturbative Quark-Gluon Plasma (pQGP). Instead, RHIC seems to probe a novel region, which has been dubbed the "sQGP" [2].

In this paper I sketch how to develop an effective theory for the sQGP. Classically, the model is a familiar spin system, a gauged principal chiral field [3]; beyond leading order, it is more involved. A mean field approximation to the effective theory gives a random matrix model [3]. Such models are dominated by eigenvalue repulsion from the Vandermonde determinant in the measure. In fact, for a $SU(\infty)$ gauge theory in a small volume, deconfinement is driven by exactly such a mechanism [4]. I indicate later how eigenvalue repulsion might arise in infinite volume, from the Higgs effect for an adjoint matrix.

By the converse of asymptotic freedom, the running QCD coupling, $\alpha_s(T) = g^2(T)/(4\pi)$, increases as the temperature decreases. Thus the most natural possibility is that in the sQGP, $\alpha_s(T)$ becomes very large as the temperature $T \to T_c^+$. A phenomenology of a strongly coupled, deconfined phase has developed [2].

A definitive value for $\alpha_s(T)$ can be obtained by matching correlation functions, for the original theory in four dimensions, with an effective theory in three dimensions [5, 6, 7, 8, 9]:

$$\begin{aligned} \mathcal{L}^{eff}_{small\ A_0}(A_i, A_0) \ &= \ \mathrm{tr}\, G_{ij}^2/2 \ + \ \mathrm{tr}\ |D_i A_0|^2 \\ &+ m_D^2 \ \mathrm{tr}\, A_0^2 \ + \ \kappa_1 \left(\mathrm{tr}\, A_0^2\right)^2 \ + \ \kappa_2\, \mathrm{tr}\, A_0^4 \ . \end{aligned} \quad (1)$$

This is the Lagrangian for a massive, adjoint scalar field, A_0, coupled to static magnetic fields, A_i: A_0 and A_i are the time like and space like components of the vector potential, G_{ij} is the non-abelian magnetic field strength, and $D_i = \partial_i - ig[A_i,]$ is the covariant derivative in the adjoint representation. Fields and couplings are normalized as in four dimensions, with the three dimensional action $\int d^3x/T$ times the Lagrangian. At leading order, integrating out the four dimensional modes generates a Debye mass for A_0, $m_D^2/T^2 \sim g^2$, and quartic couplings, κ_1 and κ_2, $\sim g^4$, with each a power series in g^2 [10, 11].

This effective theory represents an optimal resummation of perturbation theory. As such, it applies only when fluctuations in A_0 are small. Computing the pressure to four loop order, $\sim \alpha_s^3$, the results are complete up to one undetermined constant [9]. Even with the most favorable choice for this constant, however, the pressure does not agree with results from numerical simulations on the lattice below temperatures of $\sim 3T_c$ [6, 7].

These computations are done in imaginary time, where the "energies" are multiples of $2\pi T$. Thus the coupling constant $\alpha_s(T)$ runs with a scale which is of order $\sim 2\pi T$ [5]. Computations to two loop order show that even better, this mass scale is $\sim 9T$ [7]. For $T_c \sim 175$ MeV, this is ~ 1.6 GeV; at $3T_c$, it is ~ 4.7 GeV. While these mass scales are not asymptotic, neither are they in a non-perturbative regime: *e.g.*, $\alpha_s(1.6\ \mathrm{GeV}) \sim 0.28$ [7]. Hence the question becomes, why does this effective theory fail between T_c and $\sim 3T_c$, when the coupling is *not* that large?

To see how this might occur, consider a straight, thermal Wilson line,

$$L(x,\tau) = P\ e^{\,ig \int_0^\tau A_0(x,\tau')\, d\tau'} \ , \quad (2)$$

where P denotes path ordering, x is the spatial position, and τ, the imaginary time, runs from 0 to $1/T$. A closed loop is formed by wrapping all of the way around in imaginary time, $L(x, 1/T)$; as this quantity arises frequently, I denote it by $L(x)$.

The Wilson line is a matrix in color space, and so is not directly gauge invariant: under a gauge transformation $\mathcal{U}(x,\tau)$, $L(x) \to \mathcal{U}^\dagger(x, 1/T) L(x)\, \mathcal{U}(x, 0)$. The trace of the Wilson line is gauge invariant, and is the Polyakov loop in the fundamental representation. Normalizing so that this loop is one when $A_0 = 0$, then its expectation value should be near one if $gA_0/(2\pi T)$ is small. Numerical simulations of a lattice $SU(3)$ gauge theory show that while the expectation value of the renormalized triplet loop is near one at $3T_c$, this is not so when $T < 3T_c$. Without dynamical quarks, it drops to a value of ≈ 0.45

at T_c [12, 13], while its value with dynamical quarks is similar [14].

Since the triplet loop is significantly less than one between T_c and $\sim 3T_c$, in this region it is necessary to extend the program of [5, 6, 7, 8, 9] to construct an effective, three dimensional theory for arbitrary values of $gA_0/(2\pi T)$. While A_0 can be large, as the effective theory only applies for distances $\gg 1/T$, we can assume that all spatial momenta are small relative to $2\pi T$ [15, 16, 17, 18, 19, 20, 21]. This is like chiral perturbation theory, with temperature playing the role of the pion decay constant.

As a theory in three dimensions, this effective model should be invariant with respect with static gauge transformations. In addition, there are certain time dependent gauge transformations which are central in constraining the form of the effective Lagrangian. For definiteness, I take the gauge group to be $SU(N)$. Consider

$$\mathcal{U}_c(\tau) = e^{2\pi i\,\tau T\,t_N}\;,\; t_N = \mathrm{diag}(1\ldots 1, 1-N)\;; \qquad (3)$$

t_N is a traceless, diagonal $N\times N$ matrix. This is strictly periodic in imaginary time, $\mathcal{U}_c(1/T) = \mathcal{U}_c(0) = 1_N$, where 1_N is the unit matrix.

This particular transformation is related to that for a global $Z(N)$ transformation, which arises because a $SU(N)$ group has a nontrivial center [20, 21, 22, 23]. Whether or not the gauge group has a center symmetry [23], though, is irrelevant. Simply because they do not alter the boundary conditions in imaginary time, strictly periodic, but topologically non trivial [24], gauge transformations are allowed for any gauge group, coupled to arbitrary matter fields.

In four dimensions, the electric field is $D_iA_0 - \partial_0 A_i$. Under (3), diagonal elements of A_0 are shifted by a constant amount, $A_0^{diag} \to A_0^{diag} + 2\pi T t_N/g$, while off diagonal elements of A_0 and A_i rotate. Thus if $t_N \neq 1_N$, D_iA_0 changes when $A_i \neq 0$. Of course, this is compensated by the time dependent rotation of the A_i in $-\partial_0 A_i$.

This also shows that the electric field term in (2) is not invariant under (3): when $A_i \neq 0$, D_iA_0 changes when A_0^{diag} shifts, but now there are no time derivatives of A_i to compensate. Since the shift is $\sim 1/g$, this is fine at small A_0, but is unacceptable at large A_0 [25, 26, 27].

The significance of these large gauge transformations can be understood by looking at the Wilson line. Since it is a $SU(N)$ matrix, $L^\dagger(x)L(x) = 1_N$, it can be diagonalized by a unitary transformation,

$$L(x) = \Omega(x)^\dagger\, e^{i\lambda(x)}\, \Omega(x)\;. \qquad (4)$$

Here $\lambda(x)$ is a diagonal matrix, with elements λ_a, $a = 1\ldots N$. As $\det(L) = 1$, $\mathrm{tr}\,\lambda(x) = 0$, modulo 2π. Under static gauge transformations, $\mathcal{U}(x,\tau) = \mathcal{U}(x)$, the Wilson line transforms homogeneously, $L(x) \to \mathcal{U}^\dagger(x)L(x)\mathcal{U}(x)$, and Ω is gauge dependent, $\Omega(x) \to \Omega(x)\mathcal{U}(x)$ [28].

While the λ_a are invariant under static gauge transformations, under time dependent gauge transformations such as (3), they shift by by integral multiples of 2π, $\lambda \to \lambda + 2\pi t_N$ [22]. Thus transformations such as (3) ensure that the λ_a's are periodic variables.

The effective Lagrangian must respect this periodicity. This is automatic if it is constructed from the Wilson line, with eigenvalues $e^{i\lambda_a}$. What, then, to take for the effective electric field? I suggest

$$E_i(x) = T/(ig)\; L^\dagger(x) D_i L(x)\;. \qquad (5)$$

Like the original electric field, E_i transforms homogeneously under static gauge transformations; it is hermitean (and so is not $\sim D_iL$); it is center symmetric, if such a symmetry is present [22]; and lastly, it reduces to the expected form, $E_i \approx D_iA_0$, for small, static A_0. In mathematics, (5) is known as the left invariant one form of L [29].

Using the properties of path ordering, the effective electric field can be written as

$$\begin{aligned} E_i(x)/T \;=\; & \int_0^{1/T} d\tau\; L(x,\tau)^\dagger\, \partial_i A_0(x,\tau)\; L(x,\tau) \\ & -\; L(x,1/T)^\dagger\, [A_i(x), L(x,1/T)]\;. \end{aligned} \qquad (6)$$

Up to the various Wilson lines — which are, after all, phase factors in the gauge group — this is a plausible form for a gauge covariant electric field formed by "averaging" over τ.

With this E_i, to leading order in α_s the effective Lagrangian is

$$\mathcal{L}^{eff}_{classical}(A_i, L) = \;\mathrm{tr}\, G_{ij}^2/2 + (T^2/g^2)\;\mathrm{tr}\,\left|L^\dagger D_i L\right|^2\;. \quad (7)$$

This "classical" Lagrangian is that of a gauged, nonlinear sigma model [3]. The theory is non-renormalizeable, but this is of no concern, as this effective theory is valid only for distances $\gg 1/T$. On the lattice, the analogue of (7) is well known [30]. While I discussed (7) previously [18], the basic point of this paper is that (5) is, identically, the correct electric field for L. For a related linear model, see [31].

Using the decomposition of the Wilson line in (4), the electric field term is proportional to

$$\mathrm{tr}\;|D_iL|^2 \;=\; \mathrm{tr}\;(\partial_i\lambda)^2 \;+\; \mathrm{tr}\;\left|\left[\Omega\, D_i\, \Omega^\dagger, e^{i\lambda}\right]\right|^2\;. \qquad (8)$$

The first term on the right hand side is the same as for an abelian theory, where $E_i \sim e^{-i\lambda}\,\partial_i\, e^{i\lambda} \sim \partial_i\lambda$. Since $e^{i\lambda}$ is invariant under static gauge transformations, $\Omega(x)\, D_i\, \Omega^\dagger(x)$ must be as well: $\Omega(x) \to \Omega(x)\mathcal{U}(x)$ and $D_i \to \mathcal{U}^\dagger(x)\, D_i\, \mathcal{U}(x)$ [28]. Hence the second term represents the gauge invariant coupling between the electric and magnetic sectors in the non abelian effective theory.

The instanton number in four dimensions carries over directly to the effective theory. Start with a smooth, strictly periodic classical field, $A_\mu(x,\tau)$, and then transform to $A_0 = 0$ gauge. The gauge transformation which does this is just $L(x,\tau)$, (2). The instanton number is then a difference of Chern-Simons terms between $\tau = 1/T$ and 0 [15]. One can show that the instanton number equals the winding number of the Wilson line, $= \int d^3x\, \epsilon^{ijk}\, \mathrm{tr}\,(B_i B_j B_k)/(24\pi^2)$, $B_i = L^\dagger \partial_i L$, which is an integer.

I propose that this effective Lagrangian is most useful in the deconfined phase, at temperatures below that where the Lagrangian of (2) fails: that is, from $\sim 3T_c$ down to T_c, and for some region below T_c. At low temperatures, though, purely on geometric grounds it cannot suffice to include only straight Wilson lines. Eventually, Wilson lines which oscillate in time contribute, while at zero temperature, only closed loops matter.

The form of the effective theory is fixed by computing in a large but slowly varying A_0 field, and then comparing the result with the same exercise in the original theory [15, 16, 17, 18, 19, 20, 21]. The first step is to compute for constant L at one loop order. In a $U(N)$ gauge theory, (D3) of [15],

$$\mathcal{L}^{eff}_{1\ loop, \partial_i L = 0} = -\,\frac{2\,T^4}{\pi^2} \sum_{m=1}^{\infty} \frac{1}{m^4}\ \left|\mathrm{tr}\, L^m\right|^2\ . \qquad (9)$$

For constant L, the pressure is minus the value of the effective Lagrangian. For the perturbative vacuum, where $L = 1_N$, we obtain the pressure for an ideal $U(N)$ gas, $p_{ideal} = -\mathcal{L}^{eff}_{1\ loop}(1_N) = +N^2\pi^2 T^4/45$. Since each term in the series is negative, the minimum is given by the perturbative vacuum.

Matching the four dimensional theory to the effective theory is straightforward in principle, but technically involved. Terms for constant L correct (9); terms with two derivatives, of A_i and L, correct the classical action, (7), plus new terms [17, 18, 19, 20, 21]; there are also terms with four derivatives [20], *etc.*. All such terms should be expressed in terms of loops and powers of E_i and G_{ij}. Examples are given in [4, 26, 27]. As they arise from integrals in four dimensions, they form a power series in α_s.

Since it is stable to leading order in α_s, though, the perturbative vacuum remains stable order by order in perturbation theory. What, then, of confinement in a $SU(N)$ gauge theory without quarks? This is related to the breaking of a global $Z(N)$ symmetry in the deconfined phase: under a $Z(N)$ transformation, $L \to zL$, where $z = e^{2\pi i/N}$ [22]. Consider the diagonal $SU(N)$ matrix

$$L_c = \mathrm{diag}(1, z, z^2 \ldots z^{N-1})\ . \qquad (10)$$

Of the loops constructed from L_c, only those which are $Z(N)$ neutral are nonzero: if n is an integer, $\mathrm{tr}\,(L_c)^{mn} = 0$ when $m = 1\ldots(N-1)$, while $\mathrm{tr}\,(L_c)^{nN} = N$. Hence L_c might represent the confined vacuum [16, 32, 33]. However, at leading order, (9), this state has negative pressure, $= -\mathcal{L}^{eff}_{1\ loop}(L_c) = -(1-1/N^2)\pi^2 T^4/45$. Thus for any finite N, L_c is not a physical vacuum state.

It is at infinite N. As $N \to \infty$, the pressure is $\sim N^2$ in the deconfined phase, but only ~ 1 in the confined phase [4, 32, 33]. Thus the negative pressure of L_c, which is ~ 1 as $N \to \infty$, represents a correction $\sim 1/N^2$ to that of the deconfined phase. While (9) is only valid at leading order, as all traces of L_c vanish at infinite N, when $N \to \infty$, the pressure of L_c remains ~ 1, to all orders in $\alpha_s N$ [33].

At infinite N, L_c is familiar from matrix models: all eigenvalues appear, uniformly spread out [3, 4, 32]. As discussed above, although the complete effective Lagrangian is much more complicated than (7) and (9), the perturbative vacuum remains stable to all orders in perturbation theory. Thus, at least at infinite N, as $T \to T_c^+$, what drives the transition to a confining phase?

The answer is clear if space is a (very) small sphere [4]. Integrating out all non constant modes, one is left with a single integral, for the constant mode. This is equivalent to a random matrix model [3], where the Vandermonde determinant, in the measure of the integral for the constant mode, naturally produces eigenvalue repulsion, and drives the transition. In infinite volume, though, terms in the measure vanish with dimensional regularization.

To represent the non-perturbative effects which destablize the perturbative vacuum as $T \to T_c^+$, by hand I add to the effective Lagrangian

$$\mathcal{L}^{eff}_{non-pert.}(L) = +\,B_f\,T^2\ \left|\mathrm{tr}\, L\right|^2\ . \qquad (11)$$

This term is motivated by precise numerical simulations of a lattice SU(3) gauge theory without quarks [34]. I term B_f the "fuzzy bag constant": like the MIT bag constant, B, B_f is manifestly non-perturbative in origin. It has dimensions of mass squared, and so is a pure number times T_c^2. In (11) I write the simplest term of many, such as $|\mathrm{tr}\, L^2|^2$, *etc.*.

After (11) is added to the effective Lagrangian, the vacuum is no longer $L = 1_N$, where all eigenvalues are equal. Instead, the minimum of the loop potential is for some fixed value of $\mathrm{tr}\, L/N < 1$, in which some eigenvalues are unequal. This produces a Higgs phase for the adjoint field L: in perturbation theory, the diagonal magnetic gluons remain massless, while off diagonal gluons acquire masses. These masses are characteristic of an adjoint field, and involve *differences* of eigenvalues. Thus when the fluctuations in A_i and L are integrated out to one loop order (this is easiest in unitary gauge), the massive gauge fields contribute a new term, $\sim -\sum_{a,b=1}^{N}(g^2|e^{i\lambda_a} - e^{i\lambda_b}|^2)^{3/2}$. The sign is physical, and corresponds to eigenvalue repulsion. For a theory in three dimensions, this one loop calculation is not definitive. Most importantly, since near T_c magnetic glueballs are heavy [11], the Higgs effect probably just splits the masses of heavy magnetic glueballs by some amount. This is best measured from correlation functions of spatial plaquettes "split" in time, between $\tau = 0$ and $1/T$.

This paper is a small step towards the systematic construction of an effective Lagrangian for straight, thermal Wilson lines. Understanding deconfinement with this effective Lagrangian might be possible analytically at infinite N; small N surely requires numerical simulations on the lattice. An essential matter is how eigenvalue repulsion is generated in the effective theory. As I remarked above, in infinite volume terms in the measure vanish with dimensional regularization. The effective theory, however, has a physical cutoff set by the temperature, so perhaps a mean field ansatz is a reasonable first guess [13, 26, 35].

Assuredly, deconfinement provides a variety of novel and rich phase transitions for study.

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